《美国 Math League 专题讲座》

请大家耐心等待 …….

我们的讲座8点正式开始！！！

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从国际数学竞赛到海外名校：英语阅读能力的关键训练法

讲座内容：

- 为什么英语阅读是“竞赛”和“升学”的共通基础？
- 数学竞赛题目的英文理解 VS 海外名校的阅读要求。
- 如何提升英语阅读能力？

讲座时长预计：30分钟

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从国际数学竞赛到海外名校：

英语阅读能力的关键训练法

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PART

1

英语阅读是国际数学竞赛和升学的共同基础

1. 英语阅读

- 获取信息的能力
- 理解复杂文本的能力

2. 英语阅读是国际数学竞赛的基础

美国Math League历年真题展示

- 2023年数学大联盟四年级初赛个人赛
- 2018年数学大联盟四、五年级决赛个人赛
- 2024年数学大联盟四、五年级复赛

3. 英语阅读是国际升学的基础

标准化考试离不开阅读
不仅考你“能不能看懂”，更看你是否能：

- 分辨主旨与细节
- 推断言外之意
- 理解作者态度
- 改写或概括信息

英语阅读是国际数学竞赛和升学的共同基础

4. 英语阅读是国际大学课程的前置要求

- 理解不同文体的表达特点
- 快速阅读与信息筛选
- 批判性思维
- 跨文整合

5. 英语阅读训练的是“跨学科理解力”

英语阅读材料类型涉及如下：

- 小说文学类
- 科技类
- 评论类
- 历史、社会类

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2

以第二语言为目标来提升英语阅读

1. 英语标化考试分类

测试非母语者：剑桥体系、TOEFL、IELTS、Duolingo等

测试母语者：A-level、IB、AP、SSAT、SAT、GRE等

2. 英语标化考试目标

瞄准针对英语母语者的英语标化考试

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3

英语阅读在英语学习中的重要性

英语阅读在英语学习中的重要性

1) 提升词汇量

阅读是扩展词汇量最有效的方式之一

2) 增强语感与语法理解

阅读能帮助学习者在潜移默化中掌握英语语法结构

3) 提升理解力与批判性思维

阅读能锻炼逻辑分析、信息筛选和批判性思维的能力

4) 提高写作能力

输入决定输出

5) 了解英语文化与思维方式

语言背后承载着文化

6) 提升听说能力

阅读打下语言知识的基础，而这对于听、口同样重要

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4

英语阅读的类型

精读和泛读

1. 精读

每周精读1-2篇短文

文章长度300-600词

2. 泛读

每天泛读0.5-1小时

泛读原版小说、报刊、各类英文书

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英语精读的关键训练法

英语精读的关键训练法

1) 理清能力 (Clarifying Skills)

这是针对长句理解的阅读能力

例句：

While the government claimed **that** the new regulations would bolster economic resilience, critics contended **that** such measures might inadvertently stifle innovation among emerging enterprises.

2) 分辨能力 (Identifying Skills)

分辨长句中、段落中的概括与细节、观点与事实、 原因与结果

例句：

The Eiffel Tower is **the most beautiful** structure in Europe.

The Eiffel Tower is **330 meters tall**.

3) 改述能力 (Paraphrasing Skills)

用自己的话改述原句语义，既加强理解，又提升语用

例句：

In the large field which would **normally contain** horses, **there are** three **big** dogs **running wildly around**.

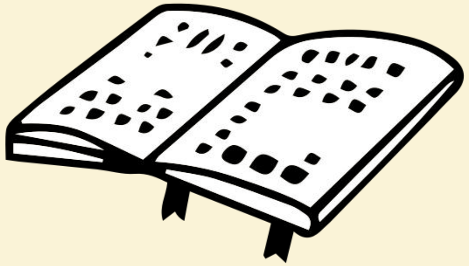
4) 推理能力 (Inferencing Skills)

A) 对生词含义的推理

B) 对语句、段落中隐含含义的推理

C) 对作者话题态度的推理

推理能力



A) 对生词含义的推理

- 并列关系带来同义词
- 转折关系带来反义词
- 细节和概括关系带来总结词

例句：

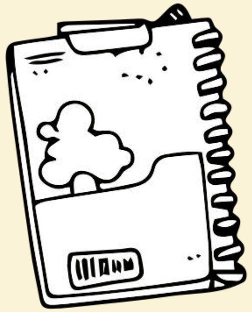
Instead of a plan of action, they continue to press for more research – a classic case of "**paralysis by analysis**."

B) 对语句、段落中隐含含义的推理

- 推理作者的言外之意要依据语句、段落中的完整细节

例句：

The current state of affairs may have been encouraged – though not justified – by **the lack of legal penalty for data leakage**.



C) 对作者话题态度的推理

- 识别褒贬义词
- 识别修辞、语气
- 识别对比、让步

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英语精读的关键训练法

5) 总结能力 (Summarizing Skills)

此能力基于分辨能力
同时要注意段落、文章中重复出现次数最多的含义

6) 提问能力 (Questioning Skills)

透过对文本内容的主动提问来促进批判性思考与深入理解

例句：

Despite the protagonist’s seemingly benevolent actions, **his ultimate motivations remained ambiguous**, casting doubt on his integrity.

7) 预测能力 (Predicting Skills)

根据已提供的叙述线索、情节走向、语气或象征，对接下来的内容或结果做出合理预测

例句：

As the narrative unfolds, the protagonist becomes **increasingly disillusioned with the political system he once served**, **foreshadowed by his growing silence and moral withdrawal**.

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感谢您的观看

Thank you

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2025-2026年度美国 Math League 全年活动安排

- **2025-2026年度初赛 (1-12年级)**

  ---日期是2025年12月06日 (暂定)，预计在2025年9月份开始报名。

- **2025-2026年度复赛 (1-10年级)**

  ---日期是2026年1月18日至2026年2月15日 (暂定)，初赛晋级后开始报名。

- **2025-2026年度决赛和数学夏令营 (1-10年级)**

  ---2026年7月，预计在2025年11月份开始报名。

- **2026美国 Math League 国际夏季数学挑战赛 (1-12年级)**

  ---日期是2026年8月16日 (暂定)，预计在2026年5月份开始报名。

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感谢聆听，感兴趣的同学联系咨询获取讲座文件，谢谢大家！

数学大联盟2023年

四年级初赛

个人赛

Question 1 (12 Credits)

Smith, Johnson, and Cohen live in Brooklyn, Manhattan, and the Bronx (not necessarily in that order). They are flying to New York City in a jet whose pilot, copilot, and navigator are named Smythe, Jenson, and Kohn (again, not necessarily respectively). It is known that

(a) Cohen lives in The Bronx.

(b) Johnson is deaf and mute.

(c) Smythe's wife and the copilot's wife are good friends.

(d) The passenger whose name sounds like the navigator's lives in Brooklyn. The navigator, however, lives in Manhattan.

(e) The navigator's next-door neighbor, one of the passengers, is a famous opera singer. Neither of the other two passengers is an opera singer.

Note:

(1) Brooklyn, Manhattan, and the Bronx are three boroughs (parts) of New York City.

(2) Smith, Johnson, Cohen, Smythe, Jenson, and Kohn are the names of the six people.

(3) Smith sounds like Smythe.

(4) Johnson sounds like Jenson.

(5) Cohen sounds like Kohn.

Questions:

(1) What is the position of Smythe? (Please enter 1 if he is the pilot, 2 if he is the copilot, and 3 if he is the navigator.)

(2) What is the position of Jenson? (Please enter 1 if he is the pilot, 2 if he is the copilot, and 3 if he is the navigator.)

(3) What is the position of Kohn? (Please enter 1 if he is the pilot, 2 if he is the copilot, and 3 if he is the navigator.)

(4) Where does Smith live? (Please enter 1 if he lives in Brooklyn, 2 if he lives in Manhattan, and 3 if he lives in the Bronx.)

(5) Where does Johnson live? (Please enter 1 if he lives in Brooklyn, 2 if he lives in Manhattan, and 3 if he lives in the Bronx.)

(6) Among the three passengers, who is the famous opera singer? (Please enter 1 if it is Simth, 2 if it is Johnson, and 3 if it is Cohen.)

Question 2 (10 Credits)

Two watches are set correctly at 7:00 AM. One watch gains 3 minutes every two hours. The other watch loses 1 minute every two hours. At what time the next day will the faster watch be exactly one hour ahead of the slower watch?

A) 9:00 AM

B) 12:00 PM

C) 1:00 PM

D) 9:00 PM

Question 3 (14 Credits)

Somewhere in the ocean, there is a very strange island where its inhabitants never make statements; they only ask questions. Then how do they manage to communicate? More on that later.

The inhabitants ask only questions answerable by yes or no. Each inhabitant is one of two types, A and B. Those of type A ask only questions whose correct answer is yes; those of type B ask only questions whose correct answer is no. For example, an inhabitant of type A could ask, "Does two plus two equal four?" But he could not ask whether two plus two equals five. An inhabitant of type B could not ask whether two plus two equals four, but he could ask whether two plus two equals five, or whether two plus two equals six.

Suppose you meet an inhabitant of this island, is it possible that he asks you, "Am I of type B?" The answer is no. If an inhabitant of type A asks, "Am I of type B?" the correct answer is no (since he isn't of type B), but a type A cannot ask any question whose correct answer is no. Therefore, no inhabitant of type A can ask this question. If an inhabitant of type B asks the question, the correct answer is yes (since he is of type B), but a type B cannot ask a question whose correct answer is yes. Therefore, an inhabitant of type B cannot ask the question either.

Suppose you meet an inhabitant of this island, he had asked you whether he is of type A. What would you have concluded? Nothing can be concluded. Any inhabitant of this island can ask whether he is of type A, because he is of either type A or type B. If he is of type A, then the correct answer to the question, "Am I of type A?" is yes, and anyone of type A can ask any question whose correct answer is yes. On the other hand, if the inhabitant is of type B, then the correct answer to the question is no, and any inhabitant of type B can ask a question whose correct answer is no.

(1) I once visited this island and met two inhabitants of this island named Ethan and Violet. I heard Ethan ask someone, "Are Violet and I both of type B?"

What type is Ethan? (Please enter 1 if he is type A, 2 if he is type B, or 0 if his type can't be decided.)

(2) I once visited this island and met two inhabitants of this island named Ethan and Violet. I heard Ethan ask someone, "Are Violet and I both of type B?"

What type is Violet? (Please enter 1 if she is type A, 2 if she is type B, or 0 if her type can't be decided.)

数学大联盟2023年
四年级初赛
个人赛

(3) I met another two inhabitants of this island named Arthur and Robert. Arthur once asked Robert, "Is at least one of us of type B?"

What type is Arthur? (Please enter 1 if he is type A, 2 if he is type B, or 0 if his type can't be decided.)

(4) I met another two inhabitants of this island named Arthur and Robert. Arthur once asked Robert, "Is at least one of us of type B?"

What type is Robert? (Please enter 1 if he is type A, 2 if he is type B, or 0 if his type can't be decided.)

(5) Next I met another two inhabitants of this island named Alice and Bob. Bob asked Alice "Are we of different types?"

What type is Alice? (Please enter 1 if she is type A, 2 if she is type B, or 0 if her type can't be decided.)

(6) Next I met another two inhabitants of this island named Alice and Bob. Bob asked Alice "Are we of different types?"

What type is Bob? (Please enter 1 if he is type A, 2 if he is type B, or 0 if his type can't be decided.)

(7) Then I met an inhabitant of this island named Zorn. He asked me, "Am I the type who could ask whether I am of type B?"

What type is Zorn? (Please enter 1 if he is type A, 2 if he is type B, or 0 if his type can't be decided.)

Question 4 (10 Credits)

Mrs. Thomas is putting down two identical red bowls and one blue bowl around a circular table to indicate seating positions for lunch. In how many ways can she arrange the red and blue bowls around the table? The answer is 1.

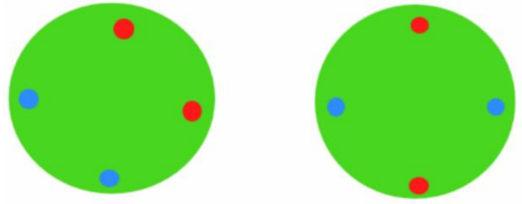
Note:

(a) Two arrangements are considered the same if one can be obtained from rotating the other one.

(b) The red and blue bowls are placed around the table with equal distance between them.

(c) Assume the table is big enough to hold all bowls.

Mrs. Thomas is putting down two identical red bowls and two identical blue bowls around a circular table to indicate seating positions for lunch. In how many ways can she arrange the red and blue bowls around the table? The answer is 2, figure below, not drawn to scale.



(1) Mrs. Thomas is putting down two identical red bowls and 5 identical blue bowls around a circular table to indicate seating positions for lunch. In how many ways can she arrange the red and blue bowls around the table?

(2) Mrs. Thomas is putting down two identical red bowls and 7 identical blue bowls around a circular table to indicate seating positions for lunch. In how many ways can she arrange the red and blue bowls around the table?

(3) Mrs. Thomas is putting down two identical red bowls and 14 identical blue bowls around a circular table to indicate seating positions for lunch. In how many ways can she arrange the red and blue bowls around the table?

(4) Mrs. Thomas is putting down two identical red bowls and 2000 identical blue bowls around a circular table to indicate seating positions for lunch. In how many ways can she arrange the red and blue bowls around the table?

数学大联盟2023年

四年级初赛

个人赛

Question 5 (10 Credits)

An anthropologist discovers an isolated tribe whose written alphabet contains only six letters (call the letters A, B, C, D, E, and F). The tribe has a taboo against using the same letter more than once in the same word. If each different sequence of letters constitutes a different word in the language, what is the maximum number of six-letter words that the language can employ?

(Note: Every six-letter word must contain exactly six letters.)

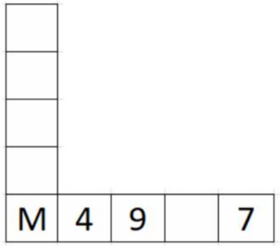
Question 6 (10 Credits)

In the figure below, each letter represents one of the digits from 0 to 9, inclusive. The same letters, for example, the two A's, stand for the same digit; different letters take different digits. A solution is the assignment of digits such that the result is a correct addition problem. What is the five-digit number represented by LATER?

```
   L E T S
+ W A V E
---------
 L A T E R
```

Question 7 (10 Credits)

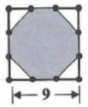
The numbers 1 through 9 are placed one per square in the figure below. The total of the 5 numbers in the horizontal row is the same as the total of the 5 numbers in the vertical column. Find all the different values that M can be.



Note: This is the way to enter your answer. For example, if M can be three values in your answer, M_1, M_2, and M_3, enter the value of $M_1 + M_2 + M_3$, similar for other answers.

Question 8 (10 Credits)

Each side of the 9 cm by 9 cm square shown below is divided into three equal parts. Find the area of the shaded region, in square cm.



Question 9 (10 Credits)

Two ranchers sold a herd of cattle and received as many dollars for each animal as there were cattle in the herd. (If there were two animals, they received 2 dollars for each one; if three, 3 dollars for each one; etc. We're told that they started with no more than 20 cattle.) With the money, they bought as many sheep as they could at 10 dollars a head, and a goat with the remainder (less than 10 dollars). Finally, they divided the animals between them. There was, however, an odd number of sheep. So one rancher took the extra sheep and the other rancher took the goat. Of course, this was not an equal division, since a sheep was worth more than a goat. How much did it cost them to buy the goat in dollars?

Question 10 (10 Credits)

Four men own the following musical instruments: Matthew, oboe and bassoon; Hank, trumpet and flute; Jack, flute and clarinet; Bill, trumpet and oboe. If the bassoon is cheaper than the oboe, the trumpet is more expensive than the flute, the oboe is cheaper than the flute, and the bassoon is more expensive than the clarinet, who owns the most expensive instruments?

(Matthew, Hank, Jack, and Bill are the names of the four men. Oboe, bassoon, trumpet, flute, and clarinet are five different musical instruments.)

Note: Please enter 1 if your answer is Matthew, 2 if your answer is Hank, 3 if your answer is Jack, and 4 if your answer is Bill.

数学大联盟2018年

四、五年级决赛

个人赛

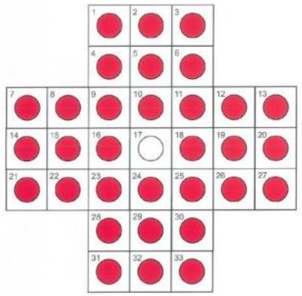
2-5. (*Time Limit: 7 minutes*) How many different two-digit numbers use two of the digits 4, 6, 8, and 9, but do not use both a 6 and a 9 in the same number?

2-6. (*Time Limit: 7 minutes*) I have 30 cards in each of 10 colors. At least how many cards must I give you to be sure I give you at least 10 cards in each of 3 different colors?

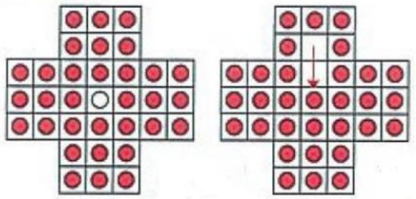
2-7. (*Time Limit: 7 minutes*) After Al and Bob ran 6 races, Al led 4 to 2. If Bob never led in the scoring, how many different scores represent one of the possible scores that lies between a 0 to 0 start and a 4 to 2 end?

26.

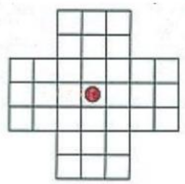
As shown below, thirty-two pegs are placed on all cells except the cell in the center (cell 17). A "jump" consists of moving a peg over any adjacent peg, removing it, and landing on the next empty cell. One is allowed to jump vertically and horizontally, but not diagonally.



For example, the first jump may be the one shown below.



The objective is to empty the entire board apart from a single peg in the central cell. That is, at the end, the board should be empty apart from a single peg in the central cell, as shown below.



(1) Is it possible to achieve this goal?

(2) How many jumps are needed to get a solution if it is possible to achieve this goal?

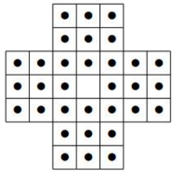
Peg Solitaire and Group Theory

Yael Algom Kfir

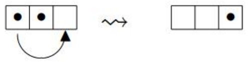
February 2006

1 A description of Cross Peg Solitaire

The board is cross shaped and contains of 35 holes. A peg might be placed in each hole. The starting configuration is a full board except for an empty place in the center.



A peg is allowed to jump vertically or horizontally over an adjacent peg to an empty hole. The peg that was jumped over is removed, decreasing the number of pegs on the board by one.

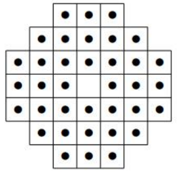


The object of the game is to invert the starting configuration, i.e. to end up with one peg in the center of the board. According to the game brochure (Milton Bradley Co., 1986), whoever succeeds in leaving the last peg in the center is a *genius*. Anyone who leaves a single peg elsewhere is an *outstanding player*.

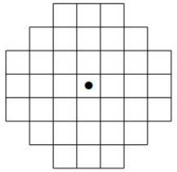
2 A variation of the game: Octagon Solitaire

We can alter the game by changing the starting position, ending position or the structure of the board. The following version of the game was manufactured and sold in Israel. The

board is now in the shape of an octagon.



The starting configuration



The winning configuration

The game's brochure didn't include a solution. Alternatively, the manufacturer offered a prize to anyone able to come up with one. No matter how generous the prize actually was, the manufacturer wasn't taking any chances by offering it. The game is insolvable!

Let's take a moment to reflect here. How can we know for sure that no matter which way we proceed we'll never be able to end up in the winning position?

In this case we can make the following rough estimate: Any peg can move in 4 directions or less. At any given time there are no more than 38 pegs on the board. Hence, at any point of the game we have no more than $4 \cdot 38 = 152$ options for legal moves. Since we lose a peg with every move and we start with 38 pegs, a single game will consist of 37 moves or less. Therefore, the number of possible games is less than $(152)^{37}$.

Therefore, in order to show that no solution exists all we have to do is check all possible games and verify that none of them is a winner. The problem is that $(152)^{37}$ is an astronomical number and it is quite impractical to go over so many games, even with the aid of a computer. Moreover, oftentimes in Math or Physics we want to affirm the impossibility of a process or the existence of an object, and there usually the number of possibilities there is not finite, so such an approach will never work.

3 Aside: No solution to the general quintic polynomial

You might have seen that the roots of the quadratic polynomial: $f(x) = ax^2 + bx + c$ are given by $x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. This means that $f(x) = 0$ only when we plug in x_1 or x_2 (when they exist). The roots x_1 and x_2 are given by a formula involving the coefficients of $f(x)$ and the operations: addition, subtraction, multiplication, division and taking square roots. The solution to the quadratic equation was probably known to the ancient babylonians as

数学大联盟2024年

四、五年级

复赛

解题时需要自己读的书

early as 1600 BC. In 1545, Girolamo Cardano, an Italian physician, published his *Ars Magna* which contained a solution to the cubic polynomial (disclosed to him by a mathematician called Fontana). This volume also contained a method, due to Ludovico Ferrari, of solving the quartic polynomial (degree 4) by reducing it to a cubic. All the formulae discovered had the striking property that the expression for the roots was built up from the coefficients by repeated addition, subtraction, multiplication, division and extraction of roots. These became known as *radical* expressions.
Since all equations of degree ≤ 4 were now solved it was natural to assume that the quintic (degree 5) equation could also be solved by a radical expression. After a long period of failed attempts and slow progress on this question, Niels Abel proved in 1824 that the general quintic equation is insolvable by radicals. In other words: **There are some polynomials of degree 5 who's roots are not radical expressions of the polynomial's coefficients!** (Clearly a counting argument could not help us here, since there is an infinite number of such expressions).
The final analysis of which polynomials are solvable by radicals and which aren't was done by a young Frenchman, Évariste Galois in 1832, though it took some 20 years for his work to be recognized. The beautiful theory he developed to solve this problem is still inspirational today in many branches of mathematics.

4 The insolvability of Octagon game

Lets "color" our board with the numbers 0 and 1 as in the figure.

		1	1	0		
	1	1	0	1	1	
1	1	0	1	1	0	1
1	0	1	1	0	1	1
0	1	1	0	1	1	0
	1	0	1	1	0	
		1	1	0		

For a configuration C let N_C be the sum of "colors" of the filled places. For example, $N^{\text{oct}}_{\text{start}} = 24$ (we do not count the empty 1 in the center).

		1	1	0		
	1	1	0	1	1	
1	1	0	1	1	0	1
1	0	1	1	0	1	1
0	1	1	0	1	1	0
	1	0	1	1	0	
		1	1	0		

Notice that preforming an elementary move might change N, or leave it unchanged as demonstrated by the following examples:

1 1 0 ⇝ 1 1 0 — N decreases by 2

1 0 1 ⇝ 1 0 1 — N doesn't change

However, the parity of N_C remains the same throughout, since moves either leave N unchanged or decrease it by 2. Let's take a moment to digest this: the parity of N_C, $p(N_C)$ is an <u>invariant</u> of the game, which means it remains constant (while C changes) because as we've seen no legal move can change it.
$N^{\text{oct}}_{\text{start}} = 24$ which is even, and since the center is marked by 1, $N^{\text{oct}}_{\text{win}} = 1$ which is odd. So we cannot get from one configuration to the other. We cannot win this game!

5 Introduction to Groups

We will now show the insolvability of the octagon game by a different method — using groups.
A group is a collection of reversible actions that we can carry out one after the other. Consider the actions you take to get dressed in the morning:

- put a sock on the right leg.
- remove sock from the left leg.
- put on both shoes.
- put on a pair of pants.

⋮

Sometimes, the order of the actions matters and sometimes it doesn't. Putting on your right sock first and then your left one, or doing it the other way around, doesn't really change the end result. However, putting on your underwear first and then your pants is different than putting on your pants first and then your underwear! It seems superman hasn't realized this yet, but he is an alien after all. Here's another example, "Don't drink and drive" is not quite the same as "Don't drive and drink".

Two elements (actions) in a group are said to <u>commute</u> if the order in which they are carried out doesn't matter. A group in which every two actions commute, is called <u>commutative</u> or <u>abelian</u> (named after Abel, whom we've already met in section 3).

6 Klein's Group

Suppose we have two light bulbs, and two buttons. Pushing button A lights the left bulb, and another push turns it off. We denote the action of pressing A by a. Pushing button B lights the right bulb, another turns it off, this will be denoted b. c is pressing the two buttons at once, and e is not doing anything at all.

Our group, K, will consist of 4 elements: a, b, c and e. Doing a and then a, denoted $a \cdot a$ or just a^2, is like doing nothing at all. We express this by the equation $a^2 = e$. The same goes for b: $b^2 = e$. Doing a and then b amounts to lighting both bulbs so $a \cdot b = c$. We completed the multiplication table for the group K below (multiplying two elements just means doing one after the other from left to right).

do second ↓ \ do first →	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Clearly, K is an abelian group.

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do second ↓ \ do first →	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Clearly, K is an abelian group.

5

mirror image, numbered (2) - just mirror image the moves!

(1)

mirror

		a	b	c		
		b	c	a		
a	b	c	a	**b**	c	a
b	c	a	b	c	a	b
c	a	b	c	a	b	c
		c	a	b		
		a	b	c		

(2)

		a	b	c		
		b	c	a		
a	b	**c**	a	b	c	a
b	c	a	b	c	a	b
c	a	b	c	a	b	c
		c	a	b		
		a	b	c		

But by our previous consideration, the configuration on the right is impossible, since it is marked by c. Hence the configuration on the left is also unattainable. We can thus rule out all one peg configurations except for the winning one and:

(i)

		a	b	c		
		b	c	a		
a	b	c	a	b	c	a
b	c	a	b	c	a	b
c	a	b	c	a	b	c
		c	a	b		
		a	b	c		

(ii)

		a	**b**	c		
		b	c	a		
a	b	c	a	b	c	a
b	c	a	b	c	a	b
c	a	b	c	a	b	c
		c	a	b		
		a	b	c		

(iii)

		a	b	c		
		b	c	a		
a	b	c	a	b	c	a
b	c	a	b	c	a	b
c	a	b	c	a	b	c
		c	a	b		
		a	**b**	c		

(iv)

		a	b	c		
		b	c	a		
a	b	c	a	b	c	a
b	c	a	b	c	a	**b**
c	a	b	c	a	b	c
		c	a	b		
		a	b	c		

Now comes the punch line: suppose we've arrived at configuration (i) then the previous one must have been:

		a	b	c		
		b	c	a		
a	b	c	a	b	c	a
b	**c**	**a**	b	c	a	b
c	a	b	c	a	b	c
		c	a	b		
		a	b	c		

But we could have easily gone from that configuration to a winning one. Therefore, whoever reaches configuration (i) must not have comprehended the rules of the game, hence must have been very lucky to end up where he did.

数学大联盟2024年

四、五年级

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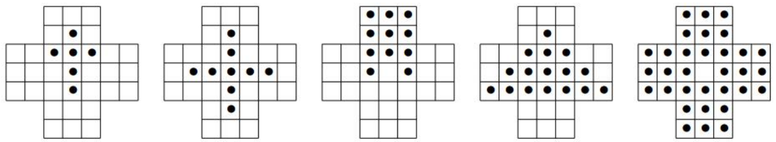
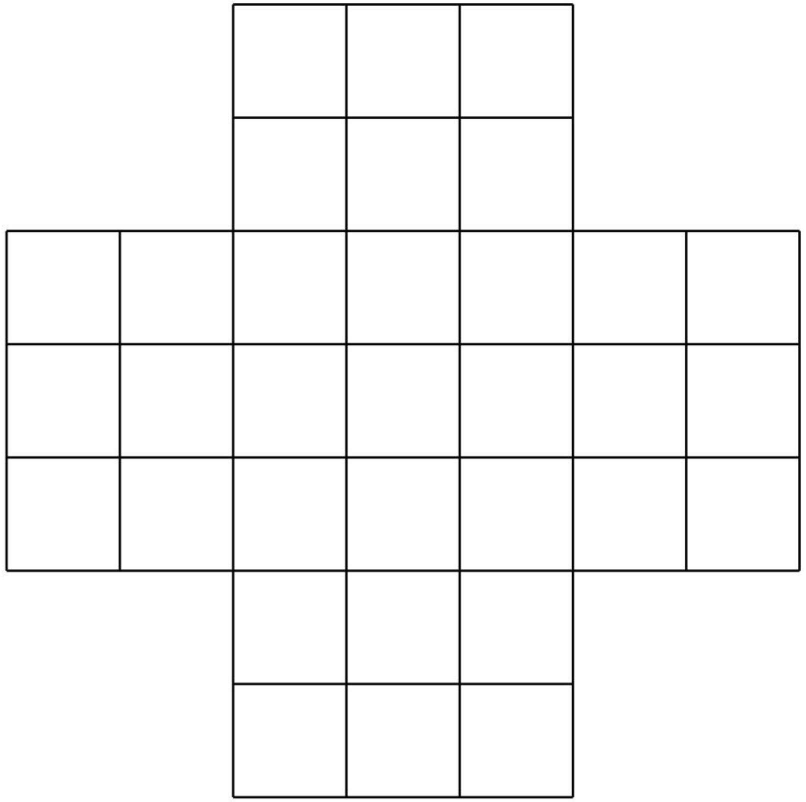
9 Warm-up

Get from the following starting positions, to the winning configuration.





10 Exercises: The First Batch

1) a. What is N_{start} for the cross shaped game with the following coloring? What is the parity of N_{start}?

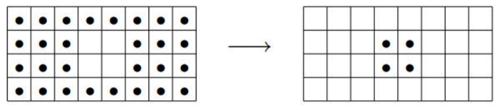
		1	1	0		
		1	0	1		
1	1	0	1	1	0	1
1	0	1	1	0	1	1
0	1	1	0	1	1	0
		0	1	1		
		1	1	0		

b. Notice that legal moves still leave $p(N_C)$ unchanged. Suppose you end up with only one peg. In which squares can it be?

2) Would the following coloring of the board still leave $p(N_C)$ invariant? Check how the contribution of a triplet changes after doing an elementary move.

		0	1	0		
		1	0	0		
0	1	0	0	1	0	0
1	0	0	1	0	0	1
0	0	1	0	0	1	0
		0	0	1		
		0	1	0		

3) Prove that the following games are insolvable.



4) Consider the board, obtained from an 8×8 chess-board by removing two corner squares

on the same diagonal. Is it possible to completely cover this board with domino pieces? Each domino piece covers two squares.

5) Can you improve the bound on the number of possible games made in section 2?

11 Exercises: A Second helping

1. Consider the following group of actions on the real line: for any integer number n, $f_n(x) = x + n$. You can think of $\mathbb{R}$ as a long (infinite) piece of string with a stationary red bead at 0. f_n shifts the string n inches to the right if n is positive and $|n|$ inches to the left if n is negative.

   a) Describe f_0.

   b) Describe $f_3 \cdot f_2$ (don't get confused this is not regular multiplication! Here $f_3 \cdot f_2$ means do f_3 and then do f_2).

   c) Describe $f_2 \cdot f_3$ (remember only the end result matters).

   d) Describe $f_1 \cdot f_{-1}$.

   e) What is $f_n \cdot f_m$ for any integer n and m?

   This group should be familiar to you by now. If we forget that the f_ns are actions and just keep in mind the way they interact in the group we get the group of integer numbers, and the multiplication between actions is simply addition of integers (Weird, isn't it?).

2. Suppose we have three drawers marked 1,2 and 3, and three balls marked a, b and c. We start with a in 1, b in 2 and c in 3. Let f be the action of switching the balls in drawers 1 and 2. Let g be the action of taking the ball in drawer 1 to drawer 2, the ball in drawer 2 to drawer 3 and the ball in drawer 3 to drawer 1.

   a) Describe $f \cdot g$.

   b) Describe $g \cdot f$.

   c) Describe f^2 and f^3.

   d) Describe g^2 and g^3.

   This is a non-abelian group, which contains 6 elements (actions). It is called: the symmetric group on 3 letters, denoted S_3.