Let’s take a look at several examples below.

让我们来看以下几个例子。

Example 1:
Problem:
How many integers between 1 and 1000 (inclusive) are divisible by 3 or 5 or 7?

## Solution:

We are asked to count the number of integers from 1 to 1000 that are divisible by **at least one** of 3, 5, or 7.

This is a classic application of the **inclusion-exclusion principle.**

Let:

- $A$ = numbers divisible by 3
- $B$ = numbers divisible by 5
- $C$ = numbers divisible by 7

We want:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Let’s compute each term:

## Step 1: Individual Counts

- $|A| = \left\lfloor \frac{1000}{3} \right\rfloor = 333$
- $|B| = \left\lfloor \frac{1000}{5} \right\rfloor = 200$
- $|C| = \left\lfloor \frac{1000}{7} \right\rfloor = 142$

### Step 2: Pairwise Intersections

- $|A \cap B| = \lfloor \frac{1000}{15} \rfloor = 66$
- $|A \cap C| = \lfloor \frac{1000}{21} \rfloor = 47$
- $|B \cap C| = \lfloor \frac{1000}{35} \rfloor = 28$

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### Step 3: Triple Intersection

- $|A \cap B \cap C| = \lfloor \frac{1000}{105} \rfloor = 9$

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### Step 4: Inclusion-Exclusion Total

$$|A \cup B \cup C| = 333 + 200 + 142 - 66 - 47 - 28 + 9 =$$

$$= 675 - 141 + 9 = 543$$

### Answer:

$$\boxed{\text{543 integers between 1 and 1000 are divisible by 3, 5, or 7.}}$$

**Why this is a strong write-up:**

1. It uses **clear notation** and organized step-by-step reasoning.
2. Demonstrates knowledge of the **inclusion-exclusion principle**.
3. Performs accurate calculations with **modular thinking**.
4. Communicates effectively without over-complication.

Example 2:

Problem:

Prove that in any group of 13 people, at least two of them will have their birthdays in the same month.

**Solution:**

There are **12 months** in a year. Imagine we have **13 people**, and we want to assign each person's birthday month to one of these 12 months.

Now apply the **Pigeonhole Principle**, which states:

> If you place more items (pigeons) into fewer categories (pigeonholes) than there are items, at least one category must contain more than one item.

In this case:

- **Pigeons** = people = 13
- **Pigeonholes** = months = 12

By the Pigeonhole Principle, if we assign 13 people to 12 months, at least **one month must contain the birthdays of at least two people.**

**Conclusion:**

$$\boxed{\text{At least two people will have their birthdays in the same month.}}$$

**Why this is a strong write-up:**

1. It clearly defines the elements involved.
2. It correctly identifies how the Pigeonhole Principle applies.
3. It concludes with a general and understandable result.
4. It uses precise language and logical structure.

Example 3:
Problem:
There are 100 people in a room. Each person shakes hands with exactly 10 others. Show that the total number of handshakes is divisible by 5.

**Solution:**

1. **Count handshakes in two ways**
   - Every person shakes hands with **10 people**.
   - There are **100 people**.
   - So if you count all the "handshakes from a person's point of view," you get:

$$100 \times 10 = 1000$$

2. **Correct the double counting**
   - But notice: when person A shakes hands with person B, **both A and B count the same handshake**.
   - So we have counted every handshake **twice**.
3. **Divide by 2**
   - The actual number of handshakes is

$$\frac{1000}{2} = 500$$

4. **Check divisibility by 5**
   - $500 \div 5 = 100$, which is a whole number.
   - So $500$ is divisible by 5.

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✅ Therefore, the total number of handshakes is **500**, and it is divisible by 5.

**Why this is a strong write-up:**
1. Clear Step-by-Step Logic
   a. It doesn't jump to the answer immediately.
   b. It shows *how* to count the handshakes and *why* we divide by 2.
2. Uses Easy Numbers
   a. Starts with 100×10 = 1000, a simple multiplication.
   b. Then divides by 2, which is easy to follow.
3. Checks the Goal Clearly
   a. It not only finds the total (500) but also explicitly shows why 500 is divisible by 5.
4. Logical Flow
   a. Count → Notice double counting → Correct → Conclude.
   b. Each step follows naturally from the one before.

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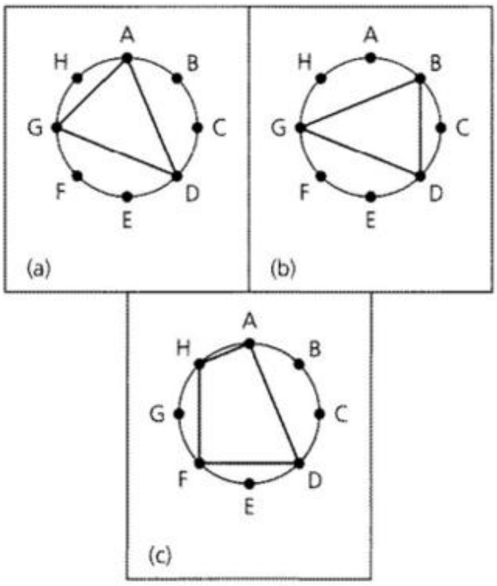
**In short:** It's a strong write-up because it is **clear, simple, and convincing** — the reader can follow the reasoning, which proves the result rigorously.

**Important:**
Please note that you don't need to write as eloquently or elegantly as the three examples above. What matters is clarity. As long as your reasoning is clear and you present it clearly, that will be sufficient. Let's look at the following two examples.

**重要:**
请注意，你不需要像上面三个例子那样写得优美或华丽。重要的是表达清晰。**只要你的推理清楚、表达清楚，就足够了。**让我们来看下面**两个例子。**

Example 4:
**Problem:**

In the three parts of the figure below, eight points are equally spaced and marked on the circumference of a given circle.




a) For parts (a) and (b), we have two different (though congruent) triangles. These two triangles (distinguished by their vertices) result from two selections of size 3 from the vertices A, B, C, D, E, F, G, H. How many different (whether congruent or not) triangles can we inscribe in the circle in this way?

b) How many different quadrilaterals can we inscribe in the circle, using the marked vertices? [One such quadrilateral appears in part (c).]

c) How many different polygons of three or more sides can we inscribe in the given circle by using three or more of the marked vertices?

**Solution:**

- Part (a):
  - To form a triangle inscribed in the circle, we need to select 3 points from all 8 point on the circle. We can find how many ways to form triangles by using the combinatio formula : $\binom{n}{m} = \frac{n!}{m!\,(n-m)!}$. In this formula, $n=8$, $m=3$.
  - The final calculation: $\binom{8}{3} = \frac{8!}{3!\,(8-3)!} = \frac{8\times7\times6\times5!}{1\times3\times2\times5!} = \frac{8\times7\times6}{3\times2\times1} = \frac{192}{6} = 56$ → two 5! canceled out
- Therefore, there are 56 ways to form triangles inscribed in the circle.

- Part (b):
  - Similar to part (a) question, to form a quadrilateral inscribed in the circle. we need to select any 4 points from all 8 points on the circle. We are asked to find how many way to form quadrilaterals are there. We can also obtain this by using the combination formula $\binom{n}{m} = \frac{n!}{m!\,(n-m)!}$. In this formula, $n=8$, $m=4$.
  - The final calculation : $\binom{8}{4} = \frac{8!}{4!\,(8-4)!} = \frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!} = \frac{8\times7\times6\times5}{4\times3\times2\times1} = \frac{1680}{24} = 70$ → two 4! canceled out
- Therefore, there are 70 ways to form quadrilaterals inscribed in the circle.

- Part (c)
  - We are asked to find all polygons with $k$ sides. $k = 3,4,5,6,7,8$, because there are 8 points on the circle in total. By using the combination formula, we know that the sum of polygons is $\binom{8}{k}$.
  - The final calculation :

$$\sum_{k=3}^{8}\binom{8}{k} = \binom{8}{3}+\binom{8}{4}+\binom{8}{5}+\binom{8}{6}+\binom{8}{7}+\binom{8}{8}$$
$$= \frac{8!}{3!\,5!} + \frac{8!}{4!\,4!} + \frac{8!}{5!\,3!} + \frac{8!}{7!\,1!} + \frac{8!}{8!\,0!}$$
$$= 56+70+56+28+8+1$$
$$= 219$$

- Therefore, there are 219 ways of forming polygons with three or more sides inscribed in the circle.

Example 5:

**Problem:**

Twelve points are placed on the circumference of a circle and all the chords connecting these points are drawn. What is the largest number of points of intersection for these chords?

**Solution:**

Solution: We are asked to find the largest number of points of intersection for these chords in the cycle.

1) To form an intersection point of chords inside the cycle, we need to select 4 distinct points out of the 12 points on the cycle. This is because for any 4 points ABCD on the cycle,the chords AC and BD (assuming the points are arranged in the order ABCD) will intersect at a unique point inside the cycle.

2) The number of way to choose 4 points out of a point is given by the combination formula, which states:

$$C(n,k)=n!/k!(n-k)!$$

Where n is the total number of the points and k is the number of points to be chosen

here

n=12 and k=4,

$\therefore$ $C(12,4)=12!/4!(12-4)!=495$

3) **In conclusion, the largest number of the points of intersection for these chords is 495.**

This test is to assess your **ability to explain mathematical reasoning clearly.**

**What we look for:**

1. Mathematical clarity and rigor
2. Communication skills
3. Original reasoning (not just memorized tricks)
4. Precision in logic, definitions, and structure

**What makes a strong write-up?**

1. Clarity: Use clear, concise language with good structure.
2. Logical flow: Start with what's known, proceed step-by-step to the conclusion.
3. Definitions: Define your variables and state any theorems you use.
4. Creativity: Some problems benefit from a clever insight — don't be afraid to be original!

本测试旨在考察你**清晰解释数学推理**的能力。

我们主要关注以下几点：

1. 数学表述的清晰性和严谨性
2. 表达与沟通能力
3. 原创性的推理 (而不仅仅是死记硬背的技巧)
4. 逻辑、定义和结构的准确性

什么样的解答算是优秀的书面说明？

1. 清晰性：使用简洁明了的语言，结构清楚。
2. 逻辑流畅：从已知条件出发，按步骤推理，最终得到结论。
3. 定义明确：清楚地定义变量，并说明你使用的定理或结论。
4. 有创意的思路：有些题目需要巧妙的想法——不要害怕展示你独特的思路！